

$\frac{2}{1,25}$ D_5

"الفاصل الضمني"

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①

Implicit Differentiation

التفاضل الضمني

Ex $y = \tan^{-1}x + (\sin x)^x$

هنا y دالة في x خفية، لذلك ان نخرج دالة
في x فقط

دكسر لو ← $y = \tan^{-1}x + e^y + x^2$

هنا تكون لا في أكثر من ^{موجوده} ولا يعكس مثل y لو موجود

ولذلك التفاضل هنا ليس تفاضل ضمني حيث قم بالتفاضل

والمجموع اي في طرف لوجوده ويصبح اي دالة في أي x

②

Ex For $\Rightarrow y^{10} = 3y^2x + \cos x^2 + e^y$

Find $\frac{dy}{dx}$

Soln $10y^9 y' = 3y^2 + x \cdot 6y y' - \sin x^2 (2x) + e^y y'$

$y' [10y^9 - 6xy - e^y] = 3y^2 - 2x \sin x^2$

$y' = \frac{3y^2 - 2x \sin x^2}{10y^9 - 6xy - e^y}$

(3)

$$\frac{1}{x} \tan(xy) = e^{x/y} + 5$$

Find $\frac{dy}{dx}$

Soln

$$\sec^2(xy) \left[y + x y' \right] = e^{x/y} \left[\frac{y - x y'}{y^2} \right] + \text{Zero}$$

$$y' \left[x \sec^2 xy + \frac{e^{x/y} x}{y^2} \right] = \frac{e^{x/y}}{y} - y \sec^2(xy)$$

$$\therefore y' = \frac{\frac{1}{y} e^{x/y} - y \sec^2(xy)}{x \sec^2 xy + \frac{x}{y^2} e^{x/y}}$$

(4)

$$[5] \quad x^{y+1} + y^{x+1} = 1$$

Soln

$$x^{y+1} \ln x \cdot y' + (y+1) x^y \cdot 1$$

$$+ y^{x+1} \ln y \cdot 1 + (x+1) y^x y' = 0$$

$$y' [x^{y+1} \ln x + y^x (x+1)] = -x^y (y+1) - y^{x+1} \ln y$$

$$y' = - \frac{x^y (y+1) + y^{x+1} \ln y}{x^{y+1} \ln x + y^x (x+1)}$$

$$3] \tan^{-1} \frac{y}{x} = \frac{1}{2} \ln(x^2 + y^2)$$

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$$\text{Soln } \frac{1}{1 + \left(\frac{y}{x}\right)^2} \left(\frac{y'}{x} + \left(\frac{-y}{x^2} \right) \right) = \frac{1}{2} \frac{2x + 2yy'}{x^2 + y^2}$$

$$\frac{y'}{1 + (y/x)^2} \cdot \frac{1}{x} - \frac{y}{x^2 + y^2} = \frac{1}{2} \left(\frac{2x}{x^2 + y^2} + \frac{2yy'}{x^2 + y^2} \right)$$

$$y' \left[\frac{1}{x} \cdot \frac{1}{1 + (y/x)^2} - \frac{y}{x^2 + y^2} \right] = \frac{x}{x^2 + y^2} + \frac{y}{x^2 + y^2}$$

$$y' = \frac{\frac{(x+y)}{x^2 + y^2}}{\frac{1}{x + y^2/x} - \frac{y}{x^2 + y^2}}$$

#

6

$$\underline{\text{Ex}} \quad y^2 + y \cos x = x^y$$

$$\underline{\text{Soln}} \quad 2y y' + \cos x (y') - y \sin x$$

$$= x^y \ln x y' + y x^{y-1} \cdot 1$$

$$y' [2y + \cos x - x^y \ln x] = y \sin x + y x^{y-1}$$

$$y' = \frac{y \sin x + x^{y-1} y}{2y + \cos x - x^y \ln x}$$

$$4] \tanh y + 2^{\tanh^{-1} x} = \sinh xy \quad (7)$$

Soln

$$\text{sech}^2 y (y') + 2^{\tanh^{-1} x} \ln 2 \frac{1}{1-x^2}$$

$$= \cosh xy (xy' + y)$$

$$y' [\text{sech}^2 y - x \cosh xy] = y \cosh xy - 2^{\tanh^{-1} x} \ln 2 \frac{1}{1-x^2}$$

$$y' = \frac{y \cosh xy - 2^{\tanh^{-1} x} \ln 2 \left(\frac{1}{1-x^2} \right)}{\text{sech}^2 y - x \cosh xy}$$

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$$(\sin x)^{\cos y} - y e^{\tan^{-1} x} = 10$$

Soln

$$(\sin x)^{\cos y} \ln(\sin x) (-\sin y) y' \quad \swarrow$$

$$+ (\cos y) (\sin x)^{\cos y - 1} \cos x - e^{\tan^{-1} x} y' \quad \swarrow$$

$$- y e^{\tan^{-1} x} \frac{1}{1+x^2} = 0$$

$$y' [-\sin y (\sin x)^{\cos y} \ln(\sin x) - e^{\tan^{-1} x}]$$

$$= y e^{\tan^{-1} x} \frac{1}{x^2 + 1} - (\cos y) (\sin x)^{\cos y - 1} \cos x$$

$$y' = \frac{y e^{\tan^{-1} x} \left(\frac{1}{x^2 + 1} \right) - (\cos y) (\sin x)^{\cos y - 1} \cos x}{-\sin y (\sin x)^{\cos y} \ln(\sin x) - e^{\tan^{-1} x}}$$

(9)

$$3] \cos^{-1}\left(\frac{x}{y}\right) = \sin(x^2 y) + 10$$

$$\begin{aligned} \underline{\text{Soln}} \quad & \frac{-1}{\sqrt{1 - \left(\frac{x}{y}\right)^2}} \left[\frac{1}{y} + \left(\frac{-x}{y^2}\right) y' \right] \\ & = \cos(x^2 y) [2xy + x^2 y'] \end{aligned}$$

$$y' \left[\frac{x}{y^2} \frac{1}{\sqrt{1 - (x/y)^2}} - x^2 \cos(x^2 y) \right] = 2xy \cos(x^2 y) + \frac{1}{y} \frac{1}{\sqrt{1 - (x/y)^2}}$$

$\therefore y' = \dots$

$$1] y^2 + y \cos x + 3^{x^2} = x^y$$

$$\begin{aligned} \underline{\text{Soln}} \quad & 2yy' + y' \cos x - y \sin x + 3^{x^2} (2x) \ln 3 = \\ & y x^{y-1} + x^y \ln x y' \end{aligned}$$

$$y' = \frac{y x^{y-1} + y \sin x - 3^x \ln 3 (2x)}{(2y + \cos x - x^y \ln x)}$$

(10)

$$7] (\sec xy)^x + \ln(\tan^{-1} x)^y = 0$$

Soln

$$(\sec xy)^x + y \ln(\tan^{-1} x)$$

$$(\sec xy)^x \ln(\sec xy) 1 + x(\sec xy)^{x-1} (\sec xy)(\tan xy)$$

$$[x y' + y] + y' \ln \tan^{-1} x + y \frac{1}{\tan^{-1} x} \frac{1}{x^2+1} = 0$$

$$y' [x^2 (\sec xy)^{x-1} \sec(xy) \tan(xy) + \ln \tan^{-1} x]$$

$$= \left[-(\sec xy)^x \ln \sec xy - \frac{y}{\tan^{-1} x} \frac{1}{x^2+1} \right]$$

$$y' = \checkmark$$